Engineering Notes

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Laminar-Sublayer Thickness in Compressible Turbulent Boundary Layers

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Nomenclature

c_f = local skin-friction coefficient at x
M = Mach number
n_1 = index of the resistance law for incompressible turbulent flows, based on log law
Pr = Prandtl number
r = recovery factor
Re = Reynolds number (based on x)
T = temperature
U = x component of velocity outside the boundary layer
x = coordinate parallel to wall
y = coordinate normal to wall
Y = thickness of laminar sublayer (based on fluid properties at the wall)
α = index of viscosity-temperature power relation
γ = isentropic exponent
μ = absolute viscosity
ν = kinematic viscosity
ρ = density
τ = local shear stress

Subscripts

i = incompressible flows
∞ = freestream
w = wall
aw = adiabatic wall (i.e., insulated wall)
l = local fluid property at the edge of laminar sublayer

Introduction

A KNOWLEDGE of the thickness of laminar sublayer is very important for the determination of maximum permissible roughness from the viewpoint of aerodynamic performance. In incompressible flows, the permissible roughness for a "hydraulically smooth" condition is estimated by applying directly the following relation[1]:

$$[(\tau_w/\rho_w)^{1/2} \cdot (y/\nu_w)] \leq 5 \tag{1}$$

Based on experiments on bodies of revolution in turbulent flows, Goddard[2] gives the following criterion for a hydraulically smooth condition (valid up to $M = 5$):

$$[(\tau_w/\rho_w)^{1/2} \cdot (y/\nu_w)] \leq 10 \tag{2}$$

A simplified analysis of the effects of compressibility and heat transfer on the characteristics of the laminar sublayer is given in Ref. 3. Even in this simplified analysis, it has not been possible to obtain explicit analytical expression for the thickness of laminar sublayer. In the present Note, a simple, explicit relation has been developed to estimate the thickness of laminar sublayer in compressible turbulent boundary layers with and without heat transfer. For the usual value of isentropic exponent equal to 1.4, the numerical results are presented as graphs to bring out quantitatively the appreciable effect of compressibility and heat transfer and for the convenience of the users.

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Analysis

A very important experimental observation[4,5] has been that at the edge of a laminar sublayer

$$[(\tau_w/\rho_w)^{1/2} \cdot (y/\nu_w)] \simeq 12 \tag{3}$$

for all Mach numbers ranging into hypersonics. Equation (3) is generally taken as the criterion for laminar sublayer thickness and is different from Eqs. (1) and (2) in respect of a constant. The thickness of laminar sublayer can be estimated from Eq. (3) by evaluating the effect of various parameters like Mach number, Reynolds number, Prandtl number, and heat transfer on various properties, namely, wall shear stress, density, and kinematic viscosity at the wall. We proceed with the analysis as follows.

Substituting for τ_w and ν_w in Eq. (3), we get

$$Y \simeq 12(2)^{1/2} \frac{\mu_w}{(c_f)^{1/2}} \cdot \frac{1}{(\rho_\infty \rho_w)^{1/2}} \cdot \frac{1}{U_\infty} \tag{4}$$

Noting that fluid properties do not change across the incompressible boundary layer and assuming that the freestream conditions remain the same in compressible and incompressible flows, Eq. (4) simplifies to (for the incompressible case)

$$Y_i \simeq 12(2)^{1/2} \frac{\mu_\infty}{(c_{fi})^{1/2}} \cdot \frac{1}{\rho_\infty} \cdot \frac{1}{U_\infty} \tag{5}$$

From Eqs. (4) and (5) we have

$$(Y/Y_i) = (c_{fi}/c_f)^{1/2} \cdot (\mu_w/\mu_\infty) \cdot (\rho_\infty/\rho_w)^{1/2} \tag{6}$$

Assuming $\mu \alpha T^\alpha$ and noting that $\rho \alpha (1/T)$ within the boundary layer, Eq. (6) can be written

$$(Y/Y_i) = (c_{fi}/c_f)^{1/2}(T_w/T_\infty)^{\alpha+1/2} \tag{7}$$

The first ratio on the right-hand side of Eq. (7) is essentially a function of freestream Mach number, wall-to-freestream temperature ratio and Prandtl number, but practically independent of Reynolds number.[6,7] The second ratio represents the effect of heat transfer and for the special case of an adia-

Table 1 Two limits for the sublayer thickness in compressible turbulent boundary layers at $Re_\infty = 10^6$

	$M_\infty = 2$ ($T_{aw}/T_\infty = 1.72$)		$M_\infty = 10$ ($T_{aw}/T_\infty = 18.9$)	
	(Y/Y_i)	(Y_l/Y_i)	(Y/Y_i)	(Y_l/Y_i)
Adiabatic wall	2.41	2	125.73	44
$T_w/T_\infty = 10$	41.95	14.70	46.66	28
$T_w/T_\infty = 1$	1.04	1.35	1.52	12.85

batic (i.e., insulated) wall, it becomes a function of Mach number and Prandtl number only, i.e.,

$$\frac{T_{aw}}{T_\infty} = \left[1 + r\left(\frac{\gamma - 1}{2}\right) M_\infty{}^2\right] \tag{8}$$

where

$$r = (Pr)^{1/3} \tag{9}$$

A simple, explicit, and analytical expression for the ratio of compressible turbulent local skin-friction coefficient valid over a wide range of Reynolds number and for general heat-transfer conditions is given in Ref. 6, further details of which may be found in Ref. 7. It is reproduced below for easy reference,

$$\frac{c_f}{c_{fi}} = \left[1 - \frac{(\alpha + 1)\log_{10}\{1 + 0.1667r[(\gamma - 1)/2]M_\infty{}^2 + 0.50(T_w/T_\infty - 1)\}}{\log_{10}Re_\infty}\right]^{-n_1} \times \left[1 + 0.1667r\left(\frac{\gamma - 1}{2}\right) M_\infty{}^2 + 0.50\left(\frac{T_w}{T_\infty} - 1\right)\right]^{-1} \tag{10}$$

Now combining Eq. (10) with Eq. (7), we get

$$\frac{Y}{Y_i} = \left[1 - \frac{(\alpha + 1)\log_{10}\{1 + 0.1667r[(\gamma - 1)/2]M_\infty{}^2 + 0.50(T_w/T_\infty - 1)\}}{\log_{10}Re_\infty}\right]^{n_1/2} \times \left[1 + 0.1667r\left(\frac{\gamma - 1}{2}\right)M_\infty{}^2 + 0.50\left(\frac{T_w}{T_\infty} - 1\right)\right]^{1/2} \left[\frac{T_w}{T_\infty}\right]^{\alpha + 1/2} \tag{11}$$

In the previous formula we have, for an adiabatic wall, $T_w/T_\infty = T_{aw}/T_\infty$, and therefore Eq. (8) can be used to determine (T_w/T_∞). It may be remarked here that for turbulent flows r is fairly well represented by Eq. (9). However, at high supersonic flows where the laminar sublayer is considerably thick compared to total boundary-layer thickness, it is reasonable to expect that the recovery factor behaves more like in laminar flows and is given by $r = (Pr)^{1/2}$.

Results and Discussion

It has been shown in Refs. 6 and 7 that Eq. (10) correlates the experimental data quite satisfactorily both for an adiabatic wall and for heat-transfer case over a wide range of Reynolds number. The ratio of laminar-sublayer thickness has been evaluated, using Eq. (11), and only a few results are shown plotted in Fig. 1, just to bring out the important features of our analysis quantitatively.

It can be seen from Fig. 1 that the effect of compressibility is to increase laminar-sublayer thickness, whereas the effect of cooling is exactly opposite in nature. To quote a few figures, for an adiabatic wall at a freestream Mach number of 10 and $Re_\infty = 10^6$, the laminar-sublayer thickness is increased to about 125 times the incompressible value. At a supersonic freestream Mach number of 2 and $Re_\infty = 10^6$ and for the case of an adiabatic wall, the laminar-sublayer thickness is increased to about $2\frac{1}{2}$ times the incompressible value. Therefore, based on the criterion of laminar-sublayer thickness, the maximum permissible value of roughness in compressible turbulent flows is increased considerably unless severe cooling is present to offset the compressibility effects, a fact that could be borne in mind by designers while specifying the tolerances on surface finish to bring down the time and cost of manufacture of various high-speed flow components. A knowledge of these effects of compressibility and heat transfer is also useful in the drag estimation of supersonic aircrafts and missiles.

The above estimate is based on the use of wall properties in the experimental criterion for laminar-sublayer thickness, viz., Eq. (3). As the turbulent eddies are affected by local fluid properties, it may be that the wall properties in Eq. (3) have to be replaced by local fluid properties. However, experimental observations in and near the laminar sublayer have been too scanty to bring out this subtle difference. An analysis of the laminar-sublayer thickness based on the use of local fluid properties in Eq. (3) would lead to the result

$$(Y_l/Y_i) = (c_{fi}/c_f)^{1/2}\cdot(T_l/T_\infty)^{\alpha+1/2} \tag{12}$$

A rough estimate for (Y_l/Y_i) can be obtained by evaluating (T_l/T_∞) using the Crocco temperature-velocity relation (which is, unfortunately, not very accurate for nonadiabatic walls) and the simple form of the law of the wall in the laminar-sublayer region. A detailed analysis of such an estimate is given in Ref. 7. It turns out that although, for adiabatic walls at low supersonic flows, the difference between (Y_l) and (Y) is not appreciable, they differ widely for nonadiabatic walls, depending on wall-to-freestream temperature ratio (see Table 1). Nevertheless, these two limits for the sublayer thickness will serve as useful guidelines in assessing maximum permissible roughness in compressible turbulent flows.

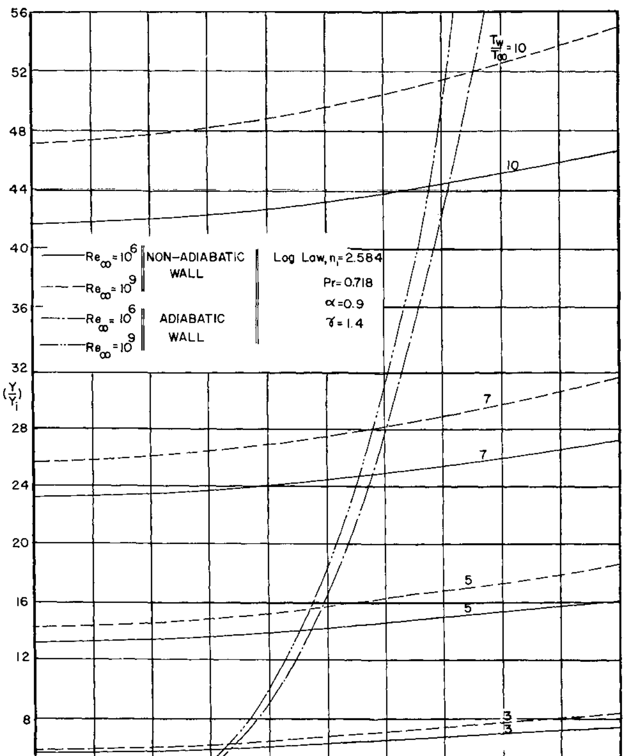

Fig. 1 Ratio (Y/Y_i) of laminar-sublayer thickness in compressible turbulent boundary layers to that in incompressible case, as a function of Mach number M_∞, Reynolds number Re_∞, and wall-to-freestream temperature ratio Tw/T_∞.

Table 2 Comparison of (Y/Y_i) based on integral approach of Baronti and Libby with those based on present analysis

				Estimated values of (Y/Y_i) based on		
Author(s)	M_∞	Re_∞	T_w/T_∞	Integral approach of Baronti and Libby	Present method using local properties	Present method using wall properties
Coles	1.982	6.18×10^6	Adiabatic wall	2.23	2.12	2.40
	2.578	8.32×10^6	"	3.25	3.07	3.65
	3.697	7.25×10^6	"	6.66	5.93	7.82
	4.554	6.83×10^6	"	11.09	9.33	13.22
Matting et al.	2.95	9.0×10^6	"	3.97	3.87	4.74
	4.20	6.20×10^6	"	8.33	7.72	10.70
	6.70[a]	7.20×10^6	"	16.54	16.29	23.53
Moore and Harkness	2.669	1.41×10^9	"	3.54	3.68	3.99
Hill	9.07	3.70×10^6	8.30	26.95	28.84	35.79
Winkler and Cha	5.24	5.11×10^6	4.97	10.43	10.78	14.60

[a] Helium data.

It is of interest to compare our estimates with those based on the integral approach of Baronti and Libby[8] for correlating velocity profiles in high-speed flows. It may be remarked here that their approach makes use of properties at the outer edge of sublayer to form the so-called sublayer Reynolds number that is assumed to be invariant. A comparison for a few selected values of experimental results (for which necessary data has been tabulated in Ref. 8) is given in Table 2. It shows that for adiabatic flows in the low supersonic range the difference between three estimates is not of any significance. However, it turns out that especially for nonadiabatic flows in the high Mach number range, the estimates based on the integral approach of Baronti and Libby agree remarkably well with those based on the present analysis using local fluid properties. This may be, perhaps, a sheer coincidence, in view of the fact that Eq. (3) is an experimental criterion based on wall properties.

Concluding Remarks

Based on experimental criterion for laminar-sublayer thickness in compressible turbulent boundary layers, a general but simple and explicit expression has been derived for predicting laminar-sublayer thickness in compressible turbulent boundary layers. The expression, which is in the form of a ratio with respect to the corresponding value in incompressible case, has been presented as an explicit function of Mach number, Reynolds number, Prandtl number, and wall-to-freestream temperature ratio. It is found that this ratio is a weak function of Reynolds number. However, it increases rapidly with Mach number but decreases with cooling (Fig. 1). The difference between the estimates based on the use of wall properties and those based on local fluid properties becomes appreciable in the high Mach number ranges and also for nonadiabatic walls.

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Active Cooling of a Hydrogen-Fueled Scramjet Engine

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Nomenclature

BL	=	boundary layer
I_{sp}	=	specific impulse, sec
T_{c_1}	=	hydrogen coolant inlet temperature, °F
T_{c_2}	=	hydrogen coolant exit temperature, °F
T_w	=	wall temperature, °F
T_{w_2}	=	wall exit temperature, °F
α	=	angle of attack, deg
η	=	heat exchanger efficiency as defined by Eq. (1)
ϕ_T	=	combustion equivalence ratio, defined as the fuel-to-air ratio required for propulsion divided by the stoichiometric fuel-to-air ratio

Introduction

ANALYTICAL studies were performed to determine engine cooling requirements for a hydrogen-fueled, scramjet powered, high-altitude Mach 12 cruise aircraft. The thermal environment requires heat protection for the internal engine walls that would otherwise reach temperatures in the order of 7500°F. Consideration of various cooling methods led to the conclusion that regenerative cooling using the hydrogen fuel as a coolant is the only feasible approach. This Note considers some of the thermal requirements and performance tradeoffs associated with the regeneratively cooled engine.

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